3

Partial Information and Signal Extraction

To this point, we have assumed that people had access to all relevant information on events up to a certain date in forming their expectations: for example $E_t x_t$ or $E(x_t \mid \Phi_t)$ implied they know the whole contents of Φ_t, the information set relating to period t and before. If they did not have access to Φ_t, then they might have access to Φ_{t-1}, the previous period's information set. However, there is an intermediate possibility, which we have up to now ignored but which has important implications for the behaviour of people with rational expectations. This possibility is that they know a part of Φ_t only, as well as all of Φ_{t-1}. For example, each individual might know the prices of certain goods which he or she trades (but not the general price level). This is 'micro' information. As for macro or aggregate information, in many economies capital market information, such as exchange rates or interest rates, is available essentially instantaneously.

When endowed with such partial knowledge, agents face a statistical inference problem. Observation of the current values of macroeconomic variables, given knowledge of the variance of disturbances in the economy, allows them to form an optimal expectation of the currently unobserved random variables using Kalman filter methods (Kalman, 1960). In particular, if a variable z is observed which is the sum of two random variables (u, e) i.e.:

$$z_t = u_t + e_t \tag{1}$$

then the current expectations of u and e are respectively given by

$$E_t(u_t) = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} z_t \tag{2}$$

$$E_t(e_t) = \frac{\sigma_e^2}{\sigma_u^2 + \sigma_e^2} z_t \tag{3}$$

where σ_u^2, σ_e^2 are the known variances of the disturbances (Graybill, 1961, has a fuller discussion). The coefficient on z_t in (2) or (3) can be thought of as that of a simple ordinary least squares regression of u_t on z_t (carried out over an infinite sample).

The purpose of this chapter is to consider two examples of how partial information alters the solution of rational expectations models; these will both illustrate the method of solution and explain the workings of two models important in their own right. The first example is the Phillips curve in Lucas' (1972b) 'islands story', where people know individual prices but not the general price level — a case of partial micro information. The second example is a general macro model where people have some capital market information — a case of partial macro information.

THE NEW CLASSICAL PHILLIPS CURVE

In the original formulation of the Phillips curve (Phillips, 1958; Lipsey, 1960), as we saw in chapter 1, short-run rigidity of wages and prices and so market disequilibrium was assumed. The curve related the change in wages to a measure of excess demand at wages that were not clearing the market. This relationship was later 'augmented' by Phelps (1970) and Friedman (1968) with the addition of expected inflation: the idea was that people were bidding up wages at a rate expected to reduce excess demand but since excess demand would only fall if wages rose in real terms, nominal wages must be bid up by an amount equal to the expected necessary rise in real wages plus the expected rise in general prices. To the resulting wage equation was usually added a price equation relating prices to costs, from which a Phillips curve in price inflation could be obtained.

The New Classical reinterpretation consists of five elements:

1. The labour market is assumed to clear: there is no wage rigidity.
2. People and firms observe prices in their own markets continuously but observe prices in other markets after a time lag. This implies that firms continuously observe the prices of their inputs and outputs, workers only their wages and some local prices (e.g. of groceries for which they continuously shop). Workers must form an expectation of the general price level.
3. Workers have rational expectations and in particular use signal extraction to infer the current general price level from the local prices they observe.

4. Workers' supply of labour has a substantial elasticity to current real wages; this is usually supported by the idea of intertemporal substitution of labour supply (although it could also occur when labour has a reservation wage for other reasons, such as unemployment benefits or 'shadow economy' earnings).

5. Firms continuously maximize profits; they relate prices to (marginal) cost, capital usually being taken as fixed, and they hire labour to the point at which its marginal value product equals its wage.

These ideas were developed by Lucas in a series of papers (Lucas and Rapping, 1969, Lucas, 1972a, b, 1973). They can be seen as a response to the difficulties of integrating rational expectations into the original Phillips curve formulation: it is hard to see why rational workers and firms should permit rigidity of wages and prices. Later, as we shall see in chapter 4, Keynesian theorists attempted to overcome this difficulty by the assumption of nominal contracts of long maturity; that assumption is not easy to reconcile with the idea of voluntary contracting by agents free to exploit all opportunities for trade, and Lucas' theory has the advantage of full consistency with that paradigm (one can also think of it as a model in which there are fully contingent wage contracts).

Let us begin with workers' intertemporal substitution. If one writes down a worker-consumer's general utility function, including terms in both consumption and leisure now and in all future periods, the first-order conditions for a maximum will among other things set the marginal rate of transformation between present and future labour supply equal to the gross real rate of interest — for an application see Sargent (1979a, chapter 16).

This is illustrated in Figure 3.1, where for simplicity the worker is assumed to have a fixed present value of consumption, $\overline{c}$, which can be achieved in a two-period life by working either this period or next: his indifference curves between leisure (work) are tangent to the trade off between minimum current and future hours required to achieve $\overline{c}$. $\overline{c}$ divided by next period's expected real wages, w^e_{t+1}, and multiplied by $1 + r$ (the gross real interest rate) gives the number of hours needed to provide $\overline{c}$ by working entirely in $t+1$ and borrowing against that income to consume in t. The tangency condition is:

$$\frac{U'_{t+1}.w^e_t}{U'_t.w^e_{t+1}} = \frac{1}{1+r}$$

where U' is the marginal utility of leisure.

Hence for a normally behaved utility function, higher current real wages will have an income effect if 'permanent' real wages rise as well, diminishing work effort, and a substitution effect (relative to future real wages), raising it. Lucas' argument is that these movements in current real wages generally leave permanent real wages unchanged and so the substitution effect is dominant, and large enough to account for the empirically observed Phillips curve correlation between prices and output.

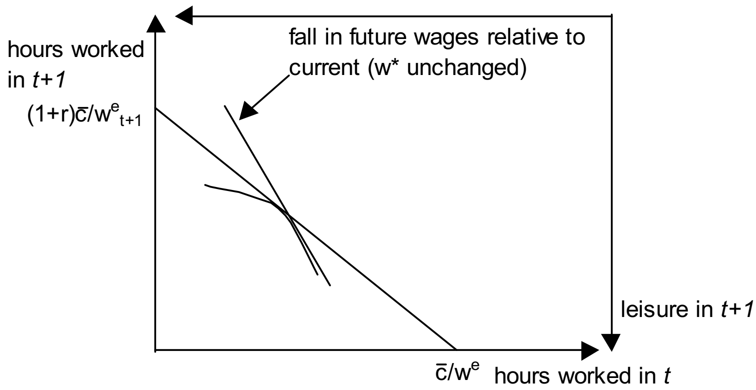


Figure 3.1: Intertemporal substitution by workers

Firms' profit maximization leads, as discussed in chapter 1, to a downward-sloping demand for labour from the first-order condition that the marginal product of labour, f', equals the real wage, or $f' = W/P$, which in the simplest case where labour is the only variable factor also expresses the price = marginal cost condition, $P = \frac{W}{f'}$; if f is a normally behaved production function, f'will fall as labour input increases.

The complete Phillips curve derivation is illustrated in figure 3.2, a four quadrant diagram due to Parkin and Bade (1988), first introduced in chapter 1 as Fig.1.1. Quadrant (a) shows the labour (L) market: the supply by workers is conditional on expected (log) prices, p^e(as well as permanent real wages, w^*, and r, both of which we hold constant here), the demand by firms depends on their own actual prices, which they continuously observe. Quadrant (b) shows the short run production function relating output ($Y = \exp y$) to labour, capital being fixed. Quadrant (c) transfers the implied output to quadrant (d), which summarizes the resulting PP relationship between (log) prices, p, and output. This is illustrated for an increase of p from $p_0 = p^e$ to p_1, with workers continuing to expect prices of $p^e = p_0$. This raises labour demand to $D'D'$ but leaves the supply curve where it is. Employment expands and more output is produced: the rise in prices has raised wages and so 'fooled' workers into thinking they are being paid higher real wages, so supplying more labour to firms who are actually enjoying lower real wage costs.

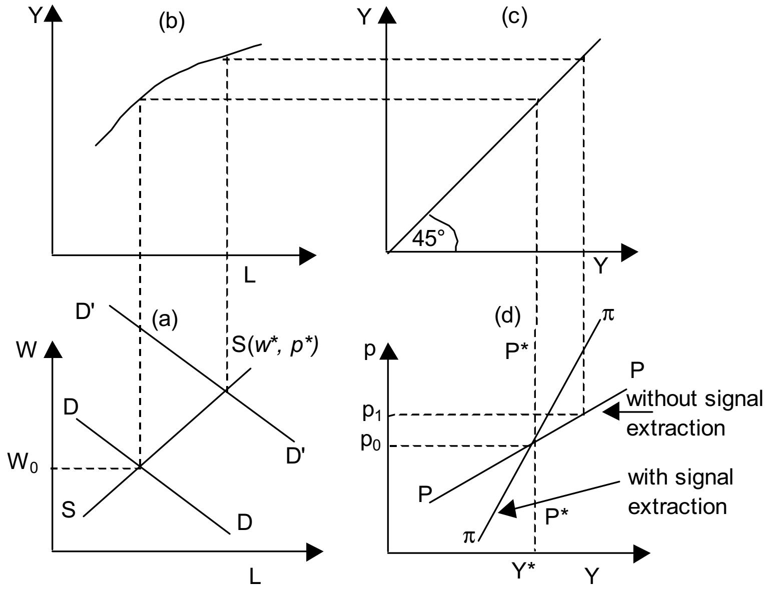


Figure 3.2: The New Classical Phillips curve

Suppose p_1 is maintained in the next period and p^e rises to this new level, then SS too shifts upwards. It will do so by exactly the same (vertical) distance as DD, namely a rise in W that is of the same proportion as the rise in p; this leaves both actual and expected real wages the same as at our starting point, and hence labour supply and demand will be the same as then. The resulting relationship between p and y, the 'long-run' Phillips curve obtained when $p = p^e$, is shown as P^*P^* and is vertical: consistently with neo-classical theory in which people and prices care about real quantities and relative prices, having no 'money illusion', rises in prices when fully anticipated or perceived have no real impact. In this simple model with no lags or adjustment in production or labour supply the economy is at its natural rate when $p = p^e$.

Let us write the resulting Phillips curve relationship between y and p (the log of respectively output and prices) as:

$$y_t - y^* = \frac{1}{\delta}(p_t - p_t^e) \tag{4}$$

This immediately brings out the formal equivalence between the traditional Phillips curve we used in chapter 2, in which prices are the dependent variable, and the New Classical formulation here in which output is. Although the derivations are clearly different, the result is formally the same.

Now let us graft on to this process the one assumption we have so far left out, signal extraction. Workers can use their current information on local prices they observe in their regular shopping to improve their expectation of the general price level. Denote the ith (of N) group of workers' (log) local price as p_{it} and assume that it is governed by

$$p_{it} = v_{it} + p_t \tag{5}$$

where v_{it} is a random error with variance of σ_v^2.

The problem faced by the ith group of workers is to forecast $p_t - E(p_t \mid \Phi_{t-1})$ from $p_{it} - E(p_{it} \mid \Phi_{t-1})$ where $\Phi_{\ t-1}$ is all last period's data assumed available to all i groups by the current period. To do this, they compute the least squares regression of $p_t - E_t(p_t \mid \Phi_{t-1})$ on $p_{it} - E(p_{it} \mid \Phi_{t-1})$ and predict from it.

We know that

$$p_t = \sum_{i=1}^{\infty} x_i \epsilon_{t-1} + p^* \tag{6}$$

$$E(p_t \mid \Phi_{t-1}) = \sum_{i=0}^{\infty} \pi_i \epsilon_{t-1} + p^* \tag{7}$$

and

$$E(p_{it} \mid \Phi_{t-1}) = E[(p_t + v_{it}) \mid \Phi_{t-1}] = E(p_t \mid \Phi_{t-1}) \tag{8}$$

since all i groups have the same $t-1$ information and cannot predict v_{it} from it. So our regression will be:

$$\pi_0 \epsilon_t = \phi_0 + \phi(\pi_0 \epsilon_t + v_{it}) \tag{9}$$

Assuming our workers had a large sample, so that we can ignore sampling error for simplicity, they will obtain (all i groups since σ_v^2 is the same for all v_{it}) $\phi_0 = 0$ because ϵ_t and v_{it} have zero means, and

$$\phi = \frac{E(\pi_0 \epsilon_t, v_{it})}{E(\pi_0 \epsilon_t, v_{it})^2} = \frac{\pi_0^2 \sigma^2}{\pi_0^2 \sigma_\epsilon^2 + \sigma_v^2}$$

Armed with this regression, the ith group's current expectation of p_t, $p_{t,i}^e = E(p_t \mid \theta_{it},\ \Phi_{t-1})$ where θ_{it} is this period's data available in the current period to the ith worker, now can be written as:

$$p_{t,i}^e = E(p_t \mid \Phi_{t-1}) + \phi(p_{it} - E[p_t \mid \Phi_{t-1}]) \tag{10}$$

Averaging expectations over all i groups gives us:

$$p_t^e = \frac{\sum_i p_{t,i}^e}{N} = (1-\phi)E(p_t \mid \Phi_{t-1}) + \phi p_t \tag{11}$$

In passing, notice that p_t^e implies an average expectation of ϵ_t. Since all i groups know that $p_t = \pi_0\epsilon_t + E(p_t \mid \Phi_{t-1})$, their expectation of ϵ_t will be given by:

$$\begin{aligned} E(\epsilon_t \mid \theta_{it}, \Phi_{t-1}) &= \frac{E(p_t \mid \theta_{it}, \Phi_{t-1}) - E(p_t \mid \Phi_{t-1})}{\pi_0} \\ &= \frac{p_{t,i}^e - E(p_t \mid \Phi_{t-1})}{\pi_0} = \frac{\phi(\, p_{it} - E(p_t \mid \Phi_{t-1})}{\pi_0} \end{aligned} \tag{12}$$

Averaging (12) over all i gives:

$$\epsilon_t^e = \frac{\phi\pi_0\epsilon_t}{\pi_0} = \phi\epsilon_t \tag{13}$$

We may now integrate p_t^e into our New Classical Phillips curve:

$$y_t - y^* = \frac{1}{\delta}(p_t - p_t^e) = \frac{(1-\phi)}{\delta}[p_t - E(p_t \mid \Phi_{t-1})] \tag{14}$$

The dashed curve in figure 3.2 shows the resulting full New Classical Phillips curve: signal extraction steepens its slope, since now as actual prices rise workers react by altering p_t^e.

Let us work out the values of ϕ and π_0 for a simple model:

$$\overline{m} + \epsilon_t = p_t + y_t \tag{15}$$

$$y_t - y* = \frac{(1-\phi)}{\delta}[p_t - E(p_t \mid \Phi_{t-1})] \tag{16}$$

Using the Muth solution we obtain: $\pi_i = 0$ $(i \geq 1)$ and $\pi_0 = \delta/(\delta + 1 - \phi)$. The slope of the Phillips curve is $\frac{1-\phi}{\delta}$ and must lie between 0 and $\frac{1}{\delta}$, since $0 < \phi < 1$.

With ϕ as given above, we can see that the solution for π_0 and ϕ is in general a cubic, with the extreme values of $\pi_0 = 1$, $\phi = 1$ (vertical Phillips curve) and $\pi_0 = \frac{\delta}{1+\delta}$, $\phi = 0$ (the PP curve in Figure 3.2).

The main implication of signal extraction is therefore that the Phillips curve's slope depends on the behaviour of monetary policy. This determines $\pi_0\epsilon_t$ in our simple models to this point; but even in more complex models where many influences impact on the 'surprise' term $(p_t - E(p_t \mid \Phi_{t-1}))$, including all demand and supply shocks, money supply shocks will still play a major role (a more complex model of this sort in which the problem is to disentangle permanent from temporary money supply shocks is set out by Brunner et al. (1980). If the money supply is volatile, then workers will assume that the current movements in prices they observe are largely monetary in origin: ϕ will be close to 1 as $\pi_0^2\sigma_\epsilon^2$ is large relative to σ_v^2 . The Phillips curve will be close to vertical. By contrast, countries with monetary stability will have flatter Phillips curves, with workers interpreting local price movements as predominantly relative price movements. Lucas (1973) turned up convincing evidence of this in a large cross-country sample of Phillips curves; and much subsequent evidence has confirmed it. One particularly obvious way in which the Phillips curve steepens is through the spread of indexation in countries with poor monetary control: indexation for a group of workers typically replaces p_t^e by a weighted average of $p_t^e$and p_t (or p_{t-k} where k is made as small as possible given information-gathering costs). Ironically, though, in countries with extreme monetary volatility indexation is handicapped both by the necessary lag in information and by poor or even fraudulent government information about the general price level.

THE SUPPLY SIDE: INTEGRATING MACRO WITH MICRO

Most of this book is concerned with the behaviour of the economy in response to monetary shocks, in the sense of shocks which disturb the absolute price level. These set up reactions through the Phillips or supply curve on output and so on other variables, for example in the way we have just examined. Yet there is a wide variety and scale of real shocks, which, regardless of their effects on the price level, cause important effects on the economy. One of the key changes in our thinking produced by the rational expectations hypothesis has been a renewed emphasis on the 'supply side'; that is, the mechanisms through which the economy responds to real shocks.

One branch of rational expectations research, real business cycle theory, dismisses monetary shocks altogether as a source of variation in output, explaining it entirely in terms of real shocks (e.g. Kydland and Prescott, 1982; Long and Plosser, 1983); these economists argue that people have sufficiently up-to-date information on the price level to avoid being fooled as in the New Classical supply curve and that any contracts they sign are fully indexed to the price level, again avoiding any real effects of unexpected price changes as in the New Keynesian supply curve. The price level on this view will vary if the money supply increases (e.g. through expansion in bank credit and deposits) but this will have no real effects. Only if there is some shock to payments technology which disturbs real plans (for example, by credit controls creating a ‘credit crunch’) will monetary shocks have a real effect: but this is not a normal money supply shock in our terms.

The real business cycle school may or may not be going too far in denying any effect of monetary shocks: testing its assertions is difficult because it can in principle account for the same correlations we observe, such as that between prices and output in the Phillips curve, by appealing to reverse causation — real shocks move output, which induces monetary expansion, which raises prices. Nevertheless, what is undoubtedly important is the focus on real shocks and the supply side as of primary interest to macroeconomists. We have just set out a basic model of the supply side, to explain the full New Classical model. This model can be developed to explain unemployment in terms of people’s voluntary choices confronted with the opportunities they face (not necessarily attractive ones of course); a further factor which may frustrate their choices however is the power of unions. Analysis of unemployment along these lines for the UK is to be found in Minford et al. (1983) and Layard and Nickell (1985), for Germany in Davis and Minford (1986) and for a variety of other countries in Bean et al. (1986). In chapter 9 we discuss these supply-side issues further; and in chapter 11 we set out a full real business cycle model.

Rational expectations has re-united macro- and microeconomics into a single subject. Keynes (1936) divided off, indeed created, the subject of macroeconomics with its own aggregate laws, not derivable from micro behaviour and subject to regular aggregate ‘market failure’. Since his intervention we have learned much about aggregate behaviour, which had never previously been much studied by the classical economists. Essentially, rational expectations has enabled us to account for macro behaviour in terms of micro laws.

CAPITAL MARKETS AND PARTIAL MACRO INFORMATION

We now consider our second example of signal extraction. Here we assume that there is no useful local information (σ_v^2 is large relative to σ_ϵ^2) but that there is current macro information from capital markets. Clearly a relevant model of most economies will contain both sorts of information but it helps our exposition to focus on each separately.

Our illustrative example supposes that people know the interest rate currently. They wish to derive from this estimates of other current macroeconomic variables — the price level and output, and so on. They do this just as in the local information case by prediction from a regression of these variables on the interest rate, any variable x_t being expressed as $x_t - E(x_t \mid \Phi_{t-1})$. Again, as in the local case, the regression parameters enter the model through their effect on the expected variables: there is an additional feedback in the model from current events to expectations, altering the impact effect of shocks to the economy.

This is a much more complicated signal extraction problem than the earlier one we considered in the Lucas supply curve where the general price level had to be extracted from the local price. There the only information people have is the local price and the equation relating this to the general price (5). Here they have information on a 'global' (economy-wide) variable, interest rates, and all the equations of the global (macro) model. Hence any view they form of the shocks driving the global interest rate must be consistent with what the model would produce; it must also be the case that the actual shocks (which will in general differ from what they expect) must via the model produce this same interest rate. This is a highly complicated consistency condition.

To get an intuitive understanding of what is happening, it is useful to consider diagrammatically how the economy behaves when this sort of signal extraction is going on; and to compare it with the usual situation we have discussed up to now where no global signal extraction is going on.

It is of some interest to compare the reaction of this 'economy', based on the model of equations (17)–(20) below, when R_t is known with that when R_t is not known. Table 3.1 shows the reactions of output to the three shocks as derived below from that model. When R_t is not known, all shocks have 'normal' positive effects on output. However, the sizes of the coefficients are quite different when R_t is known. This is hardly surprising since now output responds both to expected shocks and to the difference of shocks from their expected levels. Now even the sign of effect can be different for the various shocks. We can understand this as follows.

	u_t	v_t	e_t
y_t (R_t not known)	W	$a(\frac{c}{\alpha})W$	aW
y_t (R_t known)	$\phi_u + \frac{\phi_v}{1+a}$	$\phi_e - a\phi_u$	$a\phi_u + \frac{a\phi_v}{1+a}$
where $W = \frac{1}{1+a(1+\frac{c}{\alpha})}$			

Source: Model equations (17)–(20)

Table 3.1: Output Reactions to Shocks

Suppose the noise in e_t dominates; then $\phi_e \to 1,\ (\phi_u,\ \phi_v) \to 0$ and u_t has no effect because $E_t u_t \to 0,\ E_t v_t \to 0$. Suppose noise in v_t dominates; then $(\phi_e,\ \phi_u) \to 0$ and v_t has no effect. But suppose noise in u_t dominates; then $E_t v_t \to 0,\ \phi_u \to 1$, so that $\frac{\partial(v_t+E_t u_t)}{\partial v_t} = -a$. Hence a demand shock has a negative effect on output if supply shocks predominate, because agents misinterpret the effect of the positive demand shock on interest rates as that of a negative supply shock; expected prices consequently rise more than actual prices and supply of output is reduced.

Similar 'peculiarities' can occur in the reactions of p_t and R_t; Table 3.2 documents them. It is worth stressing therefore that the economy's behaviour in response to shocks can be 'paradoxical' if the shocks are 'misinterpreted'. Such effects are well known at the level of everyday comment (cf. the behaviour of the UK economy in l980, when the interest rates were interpreted as responding to 'overshooting' of its target by the money supply; subsequently it turned out that the money supply, truly measured, had contracted substantially). It is of interest that they can be rationalized within a stylized framework.

One such case is illustrated in figures 3.3 and 3.4. In figure 3.3, it is assumed that a negative monetary shock, e_t, occurs but that R_t is not currently known; so normal results are obtained. The LM curve shifts leftwards in the upper (R_t, y_t) half, shifting left the aggregate demand curve in the lower $(p_t,\ y_t)$ half. Prices and output fall and the interest rate rises; expectations of course are undisturbed.

	u_t	v_t	e_t
$p_t(R_t$not known)	$-(1+\frac{c}{\alpha})W$	$\frac{c}{\alpha}W$	W
$p_t(R_t$ known)	$-S$	$(1+a)S-1$	$1-aS$
$R_t(R_t$not known)	$-\frac{a}{\alpha}W$	$(1+\frac{ac}{\alpha})W/\alpha$	$-\frac{a}{\alpha}W$
$R_t(R_t$known)	A	B	D

$$S = \frac{\alpha+c}{\alpha(1+c)}\phi_u + \frac{c+\alpha(1+c)}{\alpha(1+c)(1+\alpha)}\phi_v + \frac{c(1-\mu)}{\alpha(1+c(1-\mu))}\phi_e$$

Source: Model equations (17)–(20)

Table 3.2: Price and Interest Rate Reaction to Shocks

In figure 3.4 the same shock occurs when R_t is currently observed (so that $E_t p_t$ now reacts); we illustrate the paradoxical case just discussed where people expect only variations in the supply shock, u_t, to be of any significance, so they misinterpret the shock as a negative supply shock. The left side of figure 3.4 shows actual outcomes, the right side shows expected ones: across the two sides expected outcomes are of course the same. The expected outcome is a rise in the interest rate and in prices, together with a fall in output because of a negative u_t. The rise in $E_t p_t$

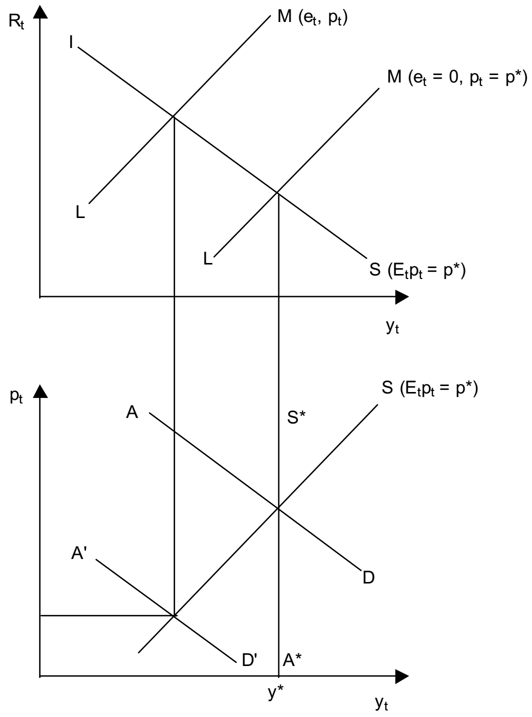
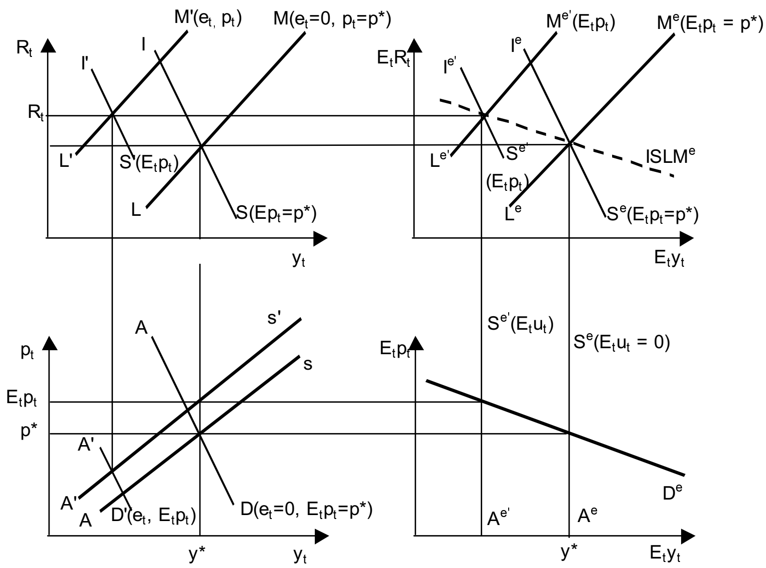


Figure 3.3: The case where R_t is not currently observed and a negative e_t shock occurs

shifts both the expected IS and LM curves, IS^e and LM^e, along the $ISLM^e$ curve which plots their intersection as Ep_t changes. In ($E_t p_t$, $E_t y_t$) space, the AS curve shifts leftwards from the fall in $E_t u_t$, along $A^e D^e$, the expected aggregate demand curve.

The actual outcome can be broken down into two parts: the shifts in the curves because $E_t p_t$ rises, and the shifts because e_t falls. The rise in $E_t p_t$ shifts the AS curve up and the AD curve leftwards in (p, y) space, the latter being produced by the IS curve shifting leftwards in (R, y) space. The fall in e_t shifts the LM leftwards in (R, y) space and so the AD leftwards in (p, y) space. These various shifts are shown on the graphs by labelling each curve according to the value of e_t, p_t and Ep_t.

In effect, the expected supply shock drives expected prices up (because of fears of shortage) so that even though prices may fall by much the same as in the normal case, the contraction in output is much greater. In this example, the misinterpretation of the monetary shock has se-



A negative ε_t shock occurs, but σ^2_v, σ^2_e are small so that a u_t shock is expected ($\phi_u \to 1$).
The ε_t shock is interpreted as a u_t shock ($E_t p_{t+1} = p^*$, i.e. $\mu = 0$)

Figure 3.4: The case where R_t is currently observed

riously worsened recession (somewhat reminiscent of the 1980 British recession).

We have seen how the economy behaves. But how in detail does one extract a signal about the shocks in this economy? First, we need to know the variance of the different shocks (we assume for simplicity that they are unrelated; if not, we also need their covariances): this tells us the unconditional probability distribution of each shock. Second, we need to know the effect each shock would have — via the model — on interest rates: with this extra knowledge we can extract the probability distribution of each shock conditional on the observed value of the interest rate, and so the conditional expected value of the shock.

Plainly, as we have seen in the diagrammatic example, the expected values of the various shocks influence the behaviour of the economy: in our example the shock expected was overwhelmingly the supply shock and this caused the economy's agents to expect a high price level when hit by a demand shock. Hence the effect of the shocks on the economy depend on the signal extraction people make; and so the effects and the signal extraction formula are set simultaneously by the structure of the model and the variances of the shocks. We now show one method for solving out what they are within the sort of model we have been using hitherto: this method has the benefit of showing transparently the interdependence between signal extraction, the model structure and the shock variances. In practical applications, however, we would recommend the use of a computer algorithm (developed in Matthews et al., 1994a, b).

To give our example enough complexity to be of interest we move to a fairly general macro model. We express all variables in deviations from equilibrium; so all constants such as $\overline{m}$ and y^* are dropped. For short, $E_t x_t$ is used to represent the current expectation conditional on last period's full data and this period's partial data (consisting of R_t, the interest rate). The model is:

$$y_t = a(p_t - E_t p_t) + u_t \tag{17}$$

$$m_t = p_t + y_t - cR_t \tag{18}$$

$$m_t = \mu m_{t-1} + e_t \tag{19}$$

$$y_t = -\alpha R_t + \alpha E_t p_{t+1} - E_t p_t + v_t \tag{20}$$

u_t, v_t and e_t are independent random shocks with known variances σ_u^2, σ_v^2 and σ_e^2; (17) is aggregate output supply; (18) is money demand; (19) is money supply; (20) is demand for aggregate output.

To solve this model, we first obtain a basic equation in the errors as deviations from their expected values. Taking expectations of (17) and subtracting the result from (17) yields:

$$y_t - Ey_t = a(p_t - E_t p_t) + u_t - E_t u_t \tag{21}$$

Equating (18) with (19) and following the same procedure as with (17) yields:

$$e_t - E_t e_t = p_t - E_t p_t + y_t - E_t y_t - c(R_t - E_t R_t) \tag{22}$$

Equation (20) analogously gives:

$$y_t - Ey_t = -(R_t - ER_t) + v_t - E_t v_t = v_t - E_t v_t \tag{23}$$

since $R_t = E_t R_t$, R_t being known. Simple manipulation then yields our basic equation in the errors:

$$(1 + a)(v_t - E_t v_t) = a(e_t - E_t e_t) + u_t - E_t u_t \tag{24}$$

This formal restriction across the expected and actual values of the errors results from the fact that people know R_t. Therefore the actual solution of R_t (based on the actual and expected errors) must coincide with the expected solution (based only on the expected errors). The parameters α and c drop out because when deviations from expectations are taken these multiply variables that must be zero (since $R_t = E_t R_t$, $E_t(E_t P_t) = E_t P_t$, $E_t(E_t P_{t+1}) = E_t P_{t+1}$).

This linear model has a general linear solution for R_t in terms of the current shocks and lagged information:

$$R_t = Au_t + Bv_t + De_t + Zm_{t-1} \tag{25}$$

A, B, D and Z are the coefficients to be solved for. Consequently, using the Graybill (1961) formula, the best estimates of u_t, v_t, e_t, given $R_t - Zm_{t-1}$ are:

$$E_t u_t = \frac{1}{A}\phi_u(Au_t + Bv_t + De_t) \tag{26}$$

$$E_t v_t = \frac{1}{B}\phi_v(Au_t + Bv_t + De_t) \tag{27}$$

$$E_t e_t = \frac{1}{D}\phi_e(Au_t + Bv_t + De_t) \tag{28}$$

where

$$\phi_u = \frac{A^2\sigma_u^2}{X}; \phi_v = \frac{B^2\sigma_v^2}{X}; \phi_e = \frac{D^2\sigma_e^2}{X}$$

and

$$X = A^2\sigma_u^2 + B^2\sigma_v^2 + D^2\sigma_e^2$$

Substituting (26), (27) and (28) into (24) we obtain:

$$(1+a)v_t - ae_t - u_t = \{(1+a)\frac{\phi v}{B} - \frac{a\phi e}{D} - \frac{\phi u}{A}\}(Au_t + Bv_t + De_t) \tag{29}$$

Since v_t, e_t and u_t may each be any real number, (3.29) is only satisfied, in the usual manner of undetermined coefficients, if the coefficients on each alone add up to zero, that is if for example

$$1 = \phi_v - \frac{aB}{(1+a)D}\phi_e - \frac{B}{(1+a)A}\phi_u \quad (\text{on } v_t) \tag{30}$$

Further, since we know that $\phi_v + \phi_e + \phi_u = 1$ it follows at once similarly that

$$\frac{D}{A} = a; \frac{D}{B} = -\frac{a}{1+a}; \frac{A}{B} = -\frac{1}{1+a} \tag{31}$$

Consequently

$$\phi_u = \frac{\sigma_u^2}{X'}; \phi_v = \frac{(1+a)^2\sigma_v^2}{X'}; \phi_e = \frac{a^2\sigma_e^2}{X'} \tag{32}$$

where

$$X' = \sigma_u^2 + (1+a)^2\sigma_v^2 + a^2\sigma_e^2$$

The full solution of this model can now be found. From (17), (18) and (19) we have:

$$p_t + a(p_t - E_t p_t) + u_t - cR_t = e_t + \mu m_{t-1} \tag{33}$$

from which we obtain, taking expectations and using $R_t = E_t R_t$:

$$R_t = \frac{1}{c}(E_t p_t + E_t u_t - E_t e_t - \mu m_{t-1}) \tag{34}$$

Equating (17) and (20), taking expectations and substituting for R_t from (34) we get:

$$E_t u_t - E_t v_t = -\frac{\alpha}{c}(E_t p_t + E_t u_t - E_t e_t - \mu m_{t-1}) - \alpha E_t p_t + \alpha E_t p_{t+1}$$

and rearranging:

$$E_t p_{t+1} - \left(\frac{1+c}{c}\right) E_t p_t = \frac{\alpha + c}{\alpha c} E_t u_t - \frac{1}{\alpha} E_t v_t - \frac{1}{c} E_t e_t - \mu/c m_{t-1} \tag{35}$$

Following Sargent's procedure (see chapter 2), we write the left hand side of (35) as

$$-\frac{1+c}{c}\left[1 - \frac{c}{1+c}B^{-1}\right] E_t p_t$$

where B is the backward operator instructing one to lag variables but not the expectations date (e.g. $BE_t P_t = E_t P_{t-1}$). Equation (35) can now be written as:

$$\begin{aligned} E_t p_t = &-\left(\frac{c}{1+c}\right)\left(\frac{1}{1 - \frac{c}{1+c}B^{-1}}\right) \\ &\left(\frac{\alpha + c}{\alpha c} E u_t - \frac{1}{\alpha} E v_t - \frac{1}{c} E\epsilon_t - \frac{\mu}{c} m_{t-1}\right) = \\ &- \frac{\alpha + c}{\alpha(1+c)} E_t u_t + \frac{c}{\alpha(1+c)} E_t v_t + \frac{1}{1+c} E_t e_t \\ &+ \frac{\mu}{1+c} \sum_{i-0}^{\infty} \left\{\frac{c}{1+c}\right\}^i E_t m_{t-1+i} \end{aligned} \tag{36}$$

Since $E_t m_{t-1+i} = m_{t-1},\ \mu m_{t-1} + E_t e_t,\ \mu^2 m_{t-1} + \mu E_t e_t,\ ...$ for $i = 0, 1, 2, ...$ (36) becomes:

$$E_t p_t = \frac{-(\alpha + c)}{\alpha(1+c)} E_t u_t + \frac{c}{\alpha(1+c)} E_t v_t + \frac{1}{1 + c(1-\mu)} E_t e_t + \frac{\mu}{1 + c(1-\mu)} m_{t-1} \quad (37)$$

whence:

$$E_t p_{t+1} = \frac{\mu^2 m_{t-1} + \mu E_t e_t}{1 + c(1-\mu)} \quad (38)$$

To find R_t substitute from (37) into (34) to obtain

$$R_t = \frac{-(1-\alpha)}{\alpha(1+c)} E_t u_t + \frac{1}{\alpha(1+c)} E_t v_t - \frac{1-\mu}{1 + c(1-\mu)} E_t e_t - \frac{\mu(1-\mu)}{1 + c(1-\mu)} m_{t-1} \quad (39)$$

We can write

$$E_t u_t = \phi_u Q_t; E_t v_t = -\frac{\phi_v}{1+a} Q_t; E_t e_t = \frac{\phi_e}{a} Q_t \quad (40)$$

where

$$Q_t = u_t - (1+a) v_t + a e_t$$

Hence

$$A = -\frac{1-\alpha}{\alpha(1+c)} \phi_u - \frac{1}{\alpha(1+c)(1+a)} \phi_v - \frac{1-\mu}{a[1 + c(1-\mu)]} \phi_e$$

$$B = -(1+a)A;$$

$$D = aA \quad (41)$$

What we see therefore in these solutions is a striking complexity in the contemporaneous response of the economy to shocks. That complexity is due to the fact that the shocks go through an 'interpretation filter' — signal extraction — which modifies their 'direct' effect, that is their effect in a world where people have no global information. Thus for as long as people do not have full information, the course of the economy is contemporaneously influenced by what they think are the shocks driving it. This influence persists after they have discovered what the shocks were, because of the lagged effects in the model: however of course this persistence also applies to the direct effects of the shocks. We can therefore describe global signal extraction as a further contemporaneous transmission mechanism of shocks, over and above their direct transmission mechanism.

CONCLUSIONS

We have seen that signal extraction has potentially important effects on economic behaviour. How people interpret shocks conditions their behaviour whether in the slope of the Phillips curve or more generally. In this chapter we have illustrated these effects and shown how models can be solved allowing for the signal extraction feedback. In subsequent chapters we shall be applying the techniques of chapters 2 and 3 to the analysis of a variety of policy issues.